

Exercise 21Calculate y' .

$$y = 3^{x \ln x}$$

SolutionCalculate y' by using the chain and product rules.

$$\begin{aligned} y' &= \frac{d}{dx}(3^{x \ln x}) \\ &= \frac{d}{dx} \left(e^{\ln 3^{x \ln x}} \right) \\ &= \frac{d}{dx} \left[e^{(x \ln x) \ln 3} \right] \\ &= e^{(x \ln x) \ln 3} \cdot \frac{d}{dx} [(x \ln x) \ln 3] \\ &= e^{\ln 3^{x \ln x}} \cdot (\ln 3) \frac{d}{dx} (x \ln x) \\ &= 3^{x \ln x} \cdot (\ln 3) \left\{ \left[\frac{d}{dx} (x) \right] \ln x + x \left[\frac{d}{dx} (\ln x) \right] \right\} \\ &= 3^{x \ln x} \cdot (\ln 3) \left[(1) \ln x + x \left(\frac{1}{x} \right) \right] \\ &= 3^{x \ln x} (\ln 3) (\ln x + 1) \end{aligned}$$